Excellent

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Birzeit University
Mathematics Department
Math 1411 Calculus I

Student Name(IN ARABIC): ....مهند مؤيد شتاك.... Student Number:..1181401..
Name of discussion teacher:....هند صبويص.... Disc. Section #:....9...
Second Exam .... First Semester 2018/2019 .... Time: 90 Minutes

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Question 1 (87%). **Choose the most correct answer:**

(1) The number of your discussion section is ..9..D

(2) The graph of the function $y = \cosh x$ is symmetric about

(a) x-axis

(b) y-axis

(c) origin

(d) none

(3) The function $f(x) = \sinh x$

(a) is concave down on $(-\infty, \infty)$ with no inflection points

(b) is concave up on $[0, \infty)$ and down on$(-\infty, 0]$ with inflection point at $x = 0$

(c) is concave up on $(-\infty, 0]$ and down on $[0, \infty)$ with inflection point at $x = 0$

(d) is concave up on $(-\infty, \infty)$ with no inflection points

(4) The domain (D) and range (R) of the function $f(x) = \tan^{-1} x$ are

tan $\tan^{-1}$

(a) $D = (\frac{-\pi}{2}, \frac{\pi}{2})$, $R = (-\infty, \infty)$

(b) $D = [-1, 1]$, $R = [-1, 1]$

(c) $D = (-\infty, \infty)$, $R = (\frac{-\pi}{2}, \frac{\pi}{2})$

(d) $D = (-\infty, \infty)$, $R = (-1, 1)$

(5) if $y = \cos(\tan^{-1}(\sqrt{x^2 - 2x}))$, $x > 2$, then

$\ln y = \ln \cos(\tan^{-1}(\sqrt{x^2-2x}))$

$\frac{y'}{y} = \frac{-\sin(\tan^{-1}(\sqrt{x^2-2x}))}{\cos(\tan^{-1}\sqrt{x^2-2x})} \cdot \frac{1}{x^2-2x+1} \cdot \frac{1}{2} \cdot \frac{x-1}{\sqrt{x^2-2x}}$

(a) $y = \dfrac{\sqrt{x^2 - 2x}}{1 - x}$

(b) $y = x - 1$

(c) $y = \dfrac{1 - x}{\sqrt{x^2 - 2x}}$

(d) $y = \dfrac{1}{x - 1}$

$= \cos(\tan^{-1}(\sqrt{x^2-2x})) \cdot (-\tan(\tan^{-1}\sqrt{x^2-2x}))(-1)$

$= \frac{-\cos(\tan^{-1}(\sqrt{x^2-2x}))}{x-1} \cdot \frac{(x-1)}{\sqrt{x^2-2x}}$

(6) $\lim\limits_{x \to 0^+} x^x =$ 0^0 $= \lim\limits_{x\to 0^+}$

$y = x^x \to \ln y = x \ln x$

$\lim\limits_{x\to 0^+} \ln y = \lim\limits_{x\to 0^+} x \ln x = \lim\limits_{x\to 0^+} \frac{\ln x}{1/x}$

$= \lim \frac{1/x}{-1/x^2} = \lim\limits_{x\to 0^+} x = 0^+$

$\lim y = \lim e^{\ln y} = \lim\limits_{x\to 0^+} e^0 = 1$

(a) ∞

(b) 0

(c) e

(d) 1

$(\frac{3}{4})^{-\infty} = \infty$ $\quad$ $0 = (\frac{5}{4})^{-\infty}$

(7) $\lim\limits_{x\to-\infty} \dfrac{4^x - 5^x}{3^x - 4^x} =$ $\lim\limits_{x\to-\infty} \dfrac{4^x\left(1 - \frac{5^x}{4^x}\right)}{4^x\left(\frac{3^x}{4^x} - 1\right)} = \dfrac{1-0}{\infty - 1} = \dfrac{1}{\infty} = 0$

(a) ∞

(b) $-\infty$

(c) 0

(d) 1

(8) one of the following is **not defined**

(a) $\cos^{-1}(\frac{\pi}{4})$

(b) $\cos^{-1}(0)$

(c) $\sin^{-1}(\frac{\pi}{2})$

(d) $\sin^{-1}(\frac{\pi}{4})$

cos

(9) $\int_0^1 \dfrac{x\,dx}{1+x^4} =$

$u = x^2$

$du = 2x \cdot dx$

$\frac{du}{2} = x \cdot dx.$

$\Rightarrow \frac{1}{2}\int_0^1 \dfrac{du}{1+u^2} = \frac{1}{2}\cdot\frac{1}{1}\cdot \tan^{-1}\left(\frac{u}{1}\right)$

$= \frac{1}{2} \tan^{-1}(x^2)\Big|_0^1 = \frac{1}{2}\tan^{-1}(1)$

$= \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}.$

(a) $\frac{\pi}{8}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $\frac{1}{8}$

$\to f' = \dfrac{+2(1-x)(+1)}{(1-x)^4}$

(10) If $f(x) = \frac{1}{(1-x)^2}, x > 1$, then $\dfrac{df^{-1}}{dx}$ at $x = \frac{1}{4}$ equals

$(f^{-1}(\tfrac{1}{4}))' = \dfrac{1}{f'(f^{-1}(b))}$

$f' = \dfrac{2(1-x)}{(1-x)^4} = \dfrac{2}{(1-x)^3}$

$(1-x)^{-2} = \to +2(1-x)^{-3}\cdot +1$

(a) $\frac{-1}{4}$

(b) 4

(c) -4

(d) $\frac{1}{4}$

(11) If $\cosh x + \sinh x = \frac{3}{4}$, then $x =$

$\dfrac{1}{4} = \dfrac{1}{(1-x)^2}$

$(1-x)^2 = 4$

$1 - x = \pm 2$

$1 - x = +2$

$\boxed{x = -1}$

$1 - x = -2$

$\boxed{x = 3} = f^{-1}(\frac{1}{4}).$

$\dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2} = \dfrac{3}{4}$

$e^x + e^{-x} + e^x - e^{-x} = \dfrac{3}{2}$

$2e^x = \dfrac{3}{2}$

$e^x = \dfrac{3}{4}$ $\boxed{x = \ln(\tfrac{3}{4})}$

(a) $\ln(\frac{2}{3})$

(b) $\ln(\frac{4}{3})$

(c) $\ln(\frac{3}{2})$

(d) $\ln(\frac{3}{4})$

$\dfrac{2}{(1-x)^3}$ $\quad (1-x)^3 = 8$ $\quad 1 - x = 2$

$f'(3) = \dfrac{2}{(1-3)^3}$

$= \dfrac{2}{(-2)^3} = \dfrac{2}{-8}$

$= \boxed{\dfrac{-1}{4}}$

(12) $\int \dfrac{3^{\sqrt{x}}}{\sqrt{x}}dx =$

(a) $(2\ln 3)3^{\sqrt{x}} + C$

(b) $(\frac{\ln 3}{2})3^{\sqrt{x}} + C$

(c) $(\frac{3}{\ln 2})3^{\sqrt{x}} + C$

(d) $(\frac{2}{\ln 3})3^{\sqrt{x}} + C$

$u = \sqrt{x}$

$du = \dfrac{1}{2\sqrt{x}}\cdot dx$

$2 \cdot du = \dfrac{dx}{\sqrt{x}}$

$\Rightarrow 2\int 3^u \cdot du = 2\cdot\dfrac{3^u}{\ln 3} + C$

$= 2\cdot\dfrac{3^{\sqrt{x}}}{\ln 3} + C$

$= \dfrac{2}{\ln 3}\cdot 3^{\sqrt{x}} + C.$

(13) If the population of a country increases exponentially with $\boxed{k = 0.1 \ln 2}$. The population will double in

(a) 5 years

(b) 10 years

(c) 20 years

(d) 2 years

$y = y_0 e^{kt}$

$2y_0 = y_0 e^{kt} \Rightarrow 2 = e^{(0.1)(\ln 2)t}$

$2 = e^{\ln(2)^{0.1t}}$

$2 = (2)^{0.1t} \rightarrow (0.1)t = 1 \Rightarrow t = \frac{1}{0.1} = 10.$

(14) $\lim\limits_{x\to 0} \dfrac{1-\cosh x}{e^x - 1} = \frac{0}{0}$

(a) 0

(b) ∞

(c) e

(d) 1

$= \lim\limits_{x\to 0} \dfrac{-\sinh x}{e^x} = \dfrac{-0}{e^0} = \dfrac{-0}{1} = 0$

(15) The statement $e^{2\ln(1-x)} = (1-x)^2$, for all $\boxed{x < 1}$ is

(a) False

(b) True

$e^{\ln(1-x)^2} = (1-x)^2 \Rightarrow (1-x)^2 = (1-x)^2$

$(1-x) > 0 \Rightarrow 1 > x \Rightarrow \boxed{x < 1}$

(16) $\lim\limits_{x\to 0} \dfrac{\sin^{-1} 5x}{x} = \frac{0}{0}$

(a) 5

(b) ∞

(c) $\frac{1}{5}$

(d) 1

$= \lim\limits_{x\to 0} \dfrac{\sin^{-1} 5x}{x} = \lim\limits_{x\to 0} \dfrac{\frac{1}{\sqrt{1-(5x)^2}} \cdot 5}{1} = \lim\limits_{x\to 0} \dfrac{5}{\sqrt{1-25x^2}}$

$= \dfrac{5}{\sqrt{1}} = 5$

(17) If $f(x) = 2 + \sqrt{\ln x}$, then $f^{-1}(x) =$

(a) $f^{-1}(x) = e^{x-2}$

(b) $f^{-1}(x) = e^{x+2}$

(c) $f^{-1}(x) = e^{(x-2)^2}$

(d) $f^{-1}(x) = e^{(x+2)^2}$

$y = 2 + \sqrt{\ln x}$

$y - 2 = \sqrt{\ln x} \Rightarrow (y-2)^2 = \ln x$

$x = e^{(y-2)^2} \Rightarrow f^{-1}(x) = e^{(x-2)^2}$

(18) $\int e^x \tanh(e^x)\, dx =$

(a) $\ln(\cosh(e^x)) + C$

(b) $\operatorname{sech}^2(e^x) + C$

(c) $\ln(\operatorname{sech}^2(e^x)) + C$

(d) $\operatorname{sech}(e^x)\tanh(e^x) + C$

$u = e^x$

$du = e^x \cdot dx$

$\Rightarrow \int \tanh(u) \cdot du = \ln(\cosh u) + C$

$= \ln(\cosh e^x) + C.$

(19) $\lim\limits_{x\to\infty} e^x 3^{-x} =$

(a) ∞

(b) $-\infty$

(c) 0

(d) $\frac{e}{3}$

$e^{\infty} \cdot 3^{-\infty} = \dfrac{e^{\infty}}{3^{\infty}} = \dfrac{\infty}{\infty}$ $(\infty \cdot 0)$

$\dfrac{3^{-x}}{e^{-x}}$

$= \lim\limits_{x\to\infty} \dfrac{e^x}{(3^x)} = \dfrac{\infty}{\infty}$

$y = e^x \cdot 3^{-x} \Rightarrow \ln y = x + \ln 3^{-x}$

$\ln y = x - x \ln 3$

$\lim\limits_{x\to\infty} \ln y = \dfrac{x^2 - x^2 \ln 3}{x}$

$= \dfrac{2x - 2x \ln 3}{1}$

$u = e^x$

$\ln u = x$

$\lim\limits_{x\to\infty}$

Please turn over

(20) $\lim_{x\to\infty}(x^2+1)^{1/x} =$

$e^0 = 1$

$\ln(x^2+1)^{\frac{1}{x}} = \frac{1}{x}\ln(x^2+1)$

$\lim_{x\to\infty} \frac{\frac{2x}{x^2+1}}{1} = \lim \frac{x}{x^2} = \frac{1}{x} = 0$

(a) 0

(b) 1

(c) e^{-1}

(d) e

(21) $\tan^{-1}(\operatorname{sech}(0)) =$ $\tan^{-1}(1) = \frac{\pi}{4}$

(a) 0

(b) -1

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

(22) The solution of the equation $\ln(\sin x) - \ln(\cos x) = 0$ in the interval $(0, \frac{\pi}{2})$ is

$\ln\left(\frac{\sin x}{\cos x}\right) = 0$

$\ln(\tan x) = 0$

$\tan x = 1$

(a) $x = \frac{\pi}{3}$

(b) $x = \frac{\pi}{6}$

(c) $x = \frac{\pi}{4}$

(d) $x = \pm\frac{\pi}{4}$

(23) If $\sin^{-1}(\cos x) = \frac{\pi}{3}$, then $x =$

$\sin^{-1}(y) = \frac{\pi}{3} \Rightarrow y = \frac{\sqrt{3}}{2} = \cos x$

$x = \frac{\pi}{6}$

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $\frac{1}{2}$

(d) $\frac{\sqrt{3}}{2}$

(24) $e^{2\ln 3} - 3^{\log_9 4} =$ $e^{\ln 9} - (9)^{\frac{1}{2}\cdot\log_9 4}$

$= 9 - (4)^{\frac{1}{2}}$

$= 9 - 2 = 7$

(a) 5

(b) 7

(c) $\frac{17}{2}$

(d) $\frac{15}{2}$

$y' = \sinh x \to (y')^2 = (\sinh x)^2$

(25) The length of the curve $\boxed{y = \cosh x}$ when $0 \le x \le \ln 2$ is

$\int_0^{\ln 2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_0^{\ln 2} \sqrt{1 + (\sinh x)^2}\, dx$

$= \int_0^{\ln 2} \sqrt{\cosh^2 x}\, dx = \int \cosh x\, dx$

$\cosh^2 - \sinh^2 x = 1$

$\cosh^2 = 1 + \sinh^2 x$

$\cosh \geq 1$

$= \{\sinh x\}\Big|_0^{\ln 2}$

$= \frac{e^x - e^{-x}}{2} = \frac{2 - \frac{1}{2}}{2} = \frac{\frac{3}{2}}{2} = \frac{3}{4}$

(a) $-\frac{1}{4}$

(b) $\frac{1}{4}$

(c) $\frac{5}{4}$

(d) $\frac{3}{4}$

(26) The function $f(x) = (\frac{1}{3})^x$ is

(a) increasing and concave up

(b) increasing and concave down

(c) decreasing and concave up

(d) decreasing and concave down

(27) The volume generated by revolving the region bounded by $y = \sqrt{x}$, the lines $y = 2$ and the y-axis about the line $y = 2$ using the washer method is given by

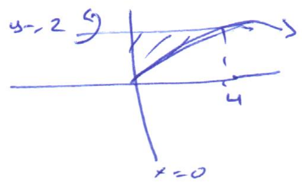
(a) $\int_0^4 \pi(2 - \sqrt{x})^2\, dx$

(b) $\int_0^4 2\pi(4 - x)\sqrt{x}\, dx$

(c) $\int_0^2 2\pi y^3\, dy$

(d) $\int_0^2 \pi(1)^2 - \pi(y^2)^2\, dy$

$V = \int_0^4 \pi\,(2 - \sqrt{x})^2 \cdot dx$



$2 = \sqrt{x}$

$x = 4$

(28) If $\sinh x = \dfrac{-3}{4}$, then $\tanh x =$

(a) $\dfrac{3}{5}$

(b) $\dfrac{-3}{5}$

(c) $\dfrac{\pm 5}{3}$

(d) $\dfrac{\pm 3}{5}$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x - \frac{9}{16} = 1$

$\cosh^2 x = \frac{25}{16}$

$\cosh x = \frac{5}{4}$

$\rightarrow \tanh x = \frac{\sinh}{\cosh} = \frac{-3/4}{5/4} = \frac{-3}{5}$

(29) The volume of the solid generated by revolving the region in the first quadrant bounded by $y = \sqrt{9 - x^2}$, $y = 0$, $x = 0$ about the y-axis equals

(a) 9π

(b) 36π

(c) 18π

(d) $\frac{16}{3}\pi$

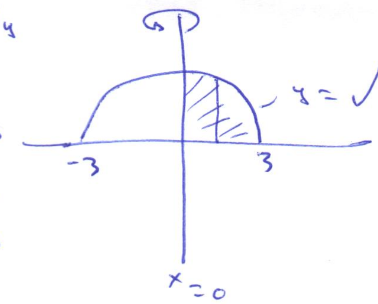
$x = \sqrt{9 - y^2}$

$V = \int_0^3 \pi\,(9 - y^2)\cdot dy$

$= \pi\,(9y - \frac{y^3}{3})\Big|_0^3$

$= \pi\,(27 - \frac{27}{3})$

$= \pi\,(27 - 9) = \boxed{\pi\, 18}$



~~$V = 2\pi \int_0^3 r h \, dx$~~

$= 2\pi \int_0^3 x\,(\sqrt{9 - x^2})\, dx = 2\pi \int_0^3 \sqrt{9x^2 - x^4}\cdot dx$

$= 2\pi \int_0^3 x \sqrt{-(x^2 - 9)}\, dx = 2\pi \int_0^3 (9x^2 - x^4)^{\frac{1}{2}} \cdot dx$

~~$= 2\pi \int_0^3 x \sqrt{\ }$~~

$= 2\pi \left[\frac{2}{3} \frac{(9x^2 - x^4)^{\frac{3}{2}}}{18x - 4x^3} \right]\Big|_0^3$

Question 2 (6%). Evaluate the following integral

$\int \frac{dx}{\sqrt{3-2x-x^2}}$ $= \int \frac{dx}{\sqrt{-(x^2+2x-3)}}$

$= \cancel{\int \frac{dx}{\sqrt{-(x+3)(x-1)}}}$

$= \int \frac{dx}{\sqrt{-(x^2+2x+1-4)}} = \int \frac{dx}{\sqrt{-((x+1)^2-4)}}$

$= \int \frac{dx}{\sqrt{4-(x+1)^2}}$ 2^2

$u = x+1$

$dx = du$

$= \sin^{-1}\left(\frac{x+1}{2}\right) + C$

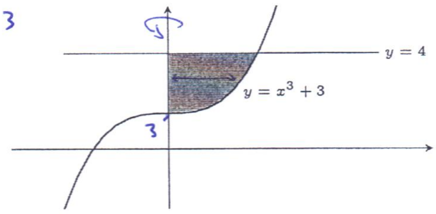
$/6$

Question 3 (16%). Consider the region in the first quadrant enclosed between $y = x^3 + 3$, $y = 4$, and y-axis.
Find the volume of the solid of revolution in the cases below. (**Do not evaluate the integrals**)

(a) The axis of revolution is the y-axis. Use the disk method.

$y = x^3 + 3$

$x = \sqrt[3]{y-3}$



$V = \int_3^4 \pi (\sqrt[3]{y-3})^2 \cdot dy$

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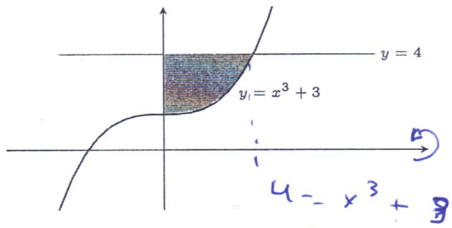
$= \frac{3}{5} \cdot \pi (y-3)^{5/3} \Big|_3^4 = \frac{3\pi}{5}(1)$

$= \frac{3\pi}{5}$

(b) The axis of revolution is the x-axis. Use the washer method.

$x^6 + 6x^3 + 9$



$V = \int_0^1 \pi ((4)^2 - (x^3+3)^2) \cdot dx$

4

$= \pi [16x - \frac{x^7}{7} - \frac{6}{4}x^4 - 9x] \Big|_0^1$

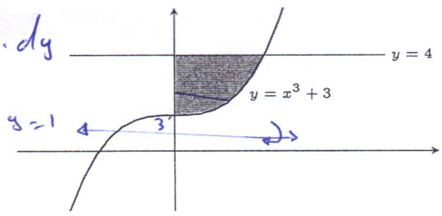
$= \pi [7 - \frac{1}{7} - \frac{6}{4}]$

$4 = x^3 + 3$

$x^3 = 1$

(c) The axis of revolution is the line $y = 1$. Use the shell method.

$V = 2\pi \int_3^4 rh \, dy = 2\pi \int_3^4 (y-1)\sqrt[3]{y-3} \cdot dy$



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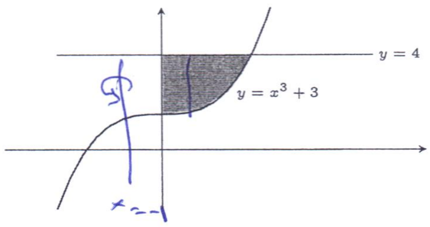
(d) The axis of revolution is the line $x = -1$. Use the shell method.

$V = 2\pi \int_0^1 (1+x)(4 - x^3 - 3) \cdot dx$

$= 2\pi \int_0^1 (1+x)(1 - x^3) \cdot dx$



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